A Nonlinear Dynamical System Approach to Birdsong Analysis

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Introduction

Songbird vocal production is a complex nonlinear phenomenon [1]. However, most acoustic studies of bird vocalization to-date rely on linear spectral analysis to characterize bird vocalizations.

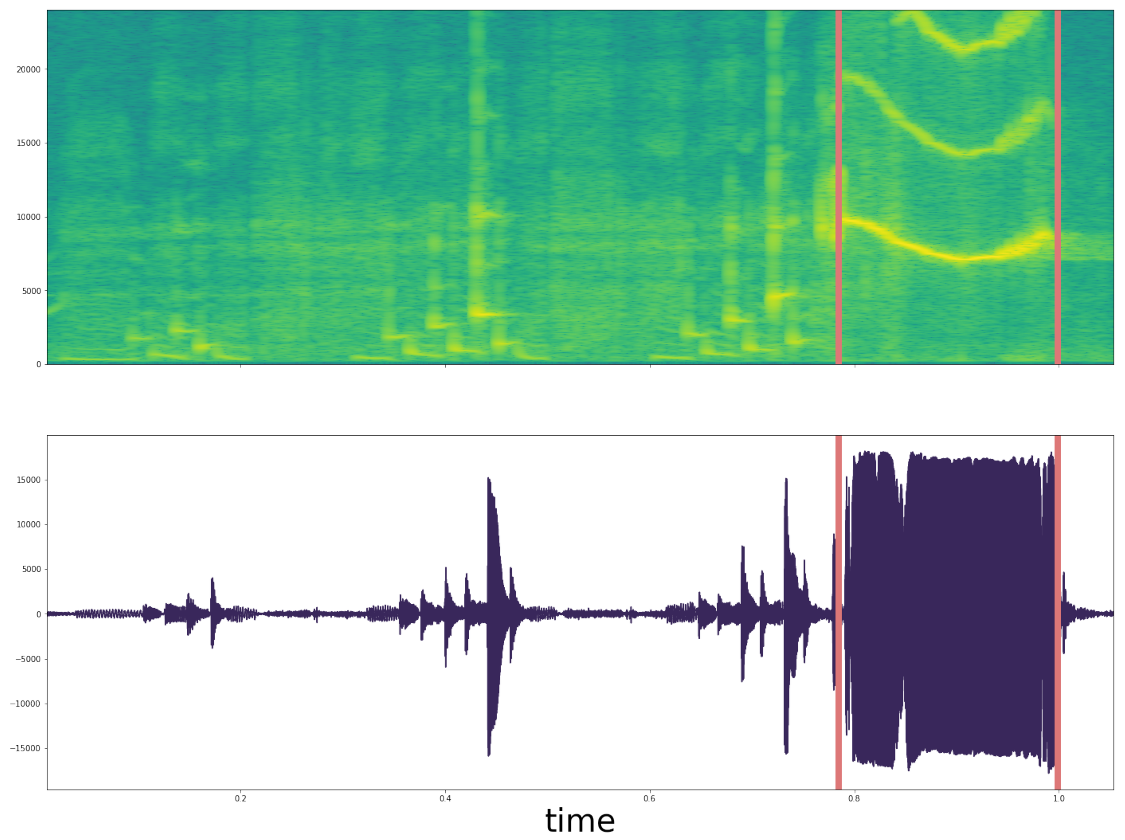

Figure 1. Spectrogram of a typical cowbird vocalization & corresponding pressure time series waveform. The time period framed in red represents the duration of analysis for the example state-space reconstruction shown in Figure 3.

Scan for Sound

Such linear analytical methods often omit the phase-dependent information of the signal. At the same time, recent studies have shown that songbirds can discriminate subtle, phase-dependent variations in calls which may encode critical information such as sex or identity [2]. We proposed a nonlinear dynamical approach to birdsong analysis. The approach could potentially illuminate previously overlooked biologically relevant information which impacts bird behaviors such as mate selection.

Methodology

The Whitney and Takens Embedding Theorems define the conditions under which it possible to reconstruct the phase space of a dynamical system given only a time series obtained from a sequence of observations of the system state [3][4]. Under these conditions, the set of all time-delayed coordinate vectors,

$$X_{D,T} = \left\{ n > n_0 : \vec{x_n}(T, D) = (x_n, x_{n-T}, ..., x_{n-(D-1)T}) \right\}$$

yields a topologically equivalent embedding of the original set, given a sufficiently large embedding dimension, D. See Figure 2 for an illustrative example.

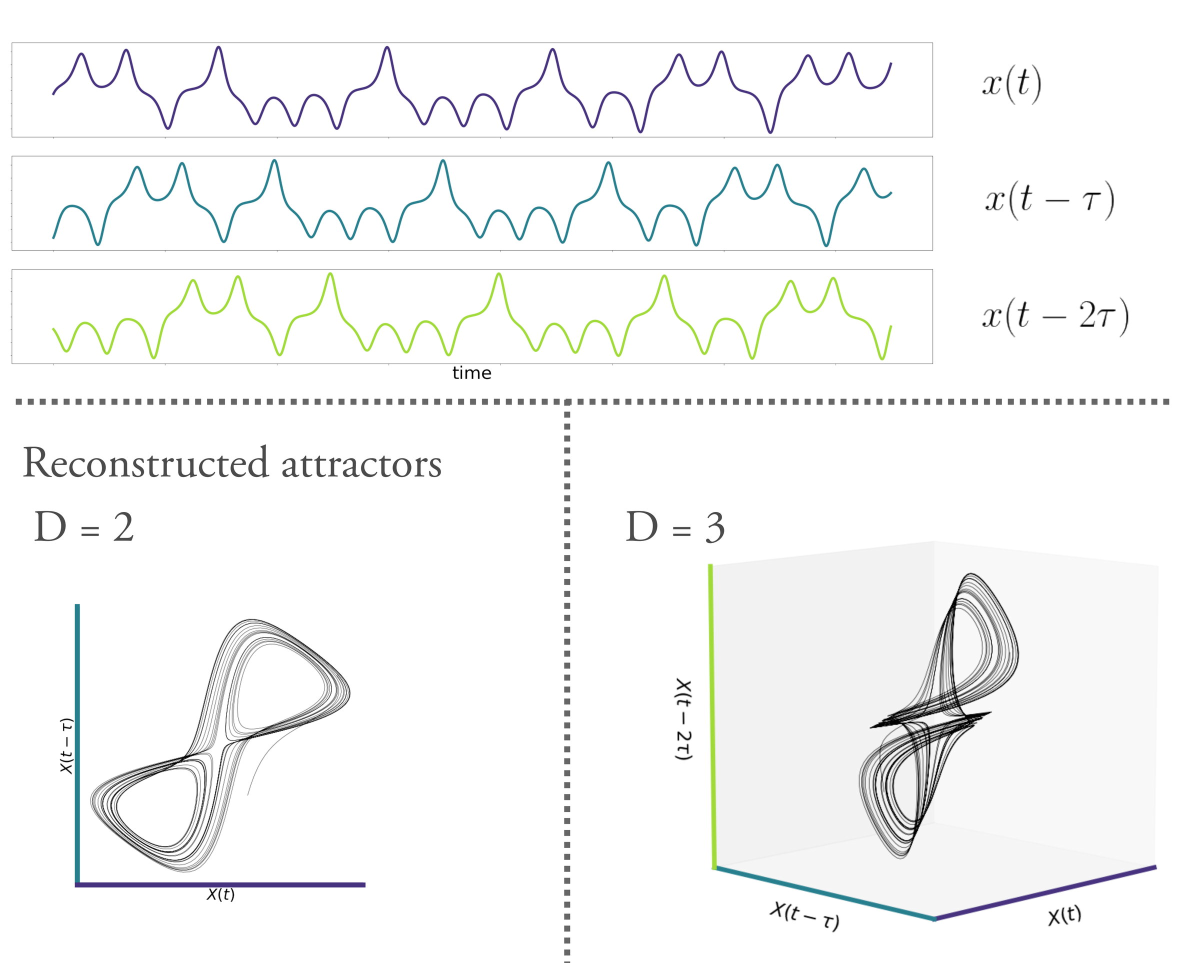

Figure 2. A reconstruction of the Lorenz system based on the time-delayed vectors of the x coordinate. The topological structure of the Lorenz attractor is preserved in the reconstruction.

Results

We created phase portraits of cowbird chirps by applying attractor reconstruction techniques on the pressure time series of the birdsongs. The embedding dimension, D, was chosen to be 3, and the time delay, T, was chosen to be 0.04 ms.

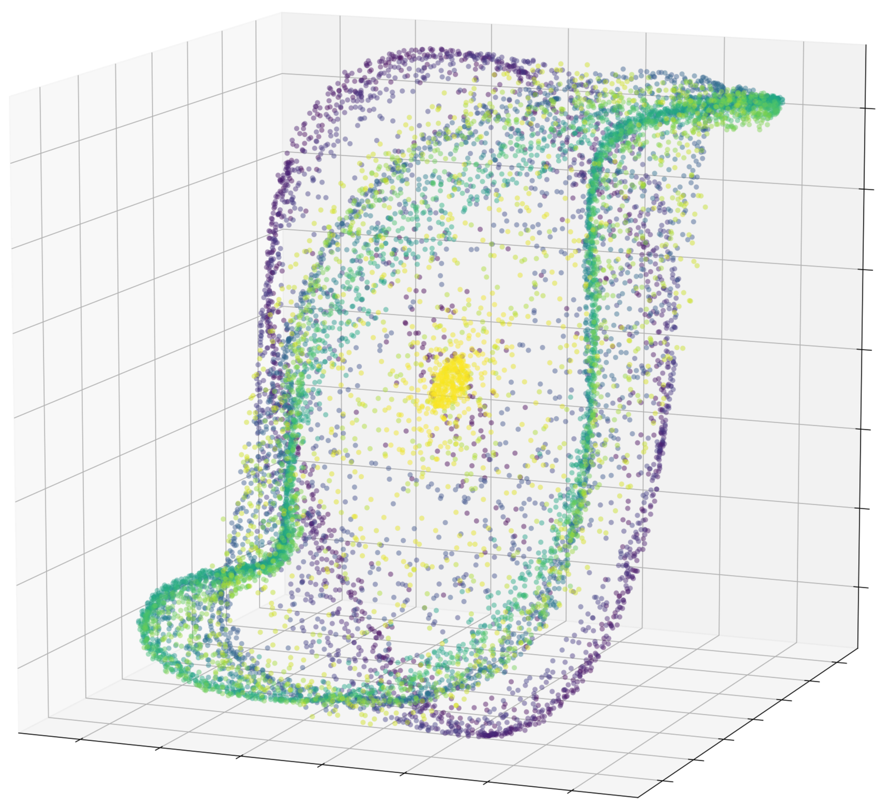

Figure 3. A phase-portrait of the cowbird vocalization segment shown in Figure 1. The state-space representation exhibits quick transitions between stable periodic limit cycles.

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In order to characterize the embedded time series, we assumed the evolution of the time-delayed vectors in the reconstructed state-space obeyed a dynamical model of the general form:

$$\frac{dX(t)}{dt} = F(X(t))$$

We assumed a second-order polynomial form for the model, and fit all reconstructed vectors in sliding windows over *X(t)*. The coefficients obtained from the polynomial fitting were then used to compactly describe the time evolution of Y(t).

We compared the coefficients with physiological parameters derived from an existing model of the avian syrinx [1], which describes the dynamics of labial position (x(t)) as driven by two time-dependent parameters: the air-sac pressure (α(t)) and the tension of labia (β(t)).

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -\alpha(t)\gamma^2 - \beta(t)\gamma^2 x - \gamma^2 x^3 - \gamma x^2 y + \gamma^2 x^2 - \gamma x y$$

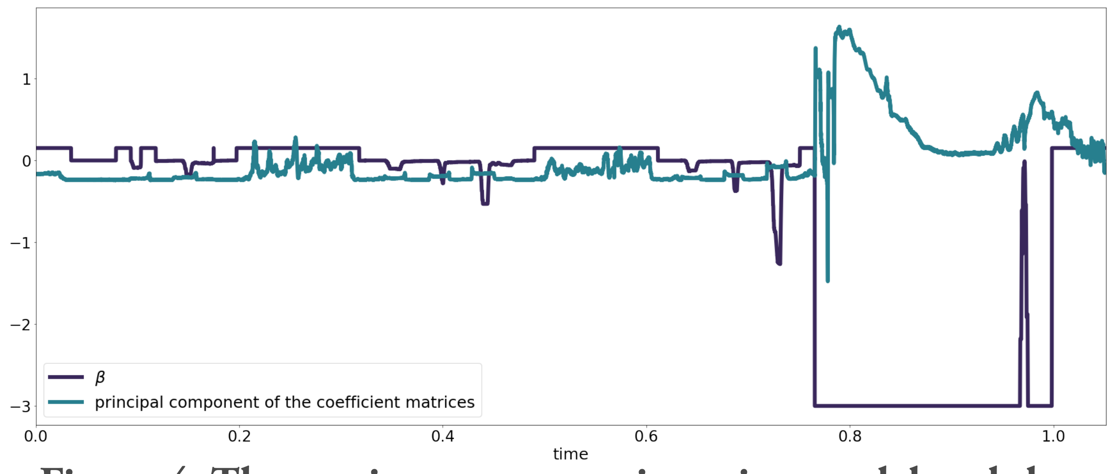

Figure 4. The tension parameter in syrinx model and the primary component of the coefficient matrices. The two statistics show strong correlation.

Conclusion

We obtained encouraging results indicating transitions in state-space trajectories of birdsong pressure time series might directly correspond to transitions in the states of the underlying sound production system. In future studies, we plan to further explore these potential connections with empirical data.

References

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[2] Prior et al. Scientific Reports 8, 6212 (2018)
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